TEMPERATURE FIELD OF AN ICE SHELF IN THE VICINITY OF A HOT WATER-DRILLED WELL

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The temperature of an ice shelf following open thermic drilling is investigated. The time dependence of the ice temperature and the profile of its lower surface are analyzed. The time of relaxation of the system ocean–ice shelf–atmosphere to the equilibrium state is estimated. The results are used to substantiate the use of data on the dynamics of the lower surface of an ice shelf in monitoring of global temperature of the ocean.

Introduction. Ice shelves can be used for global monitoring of the ocean temperature. As is well known [1, 2], a layer with a thickness up to several tens of centimeters melts or freezes on the lower surface of an ice shelf annually. Estimates show that even an insignificant (of the order of a few thousandths of a degree) increase or decrease in the ocean temperature leads to additional ablation or accumulation of ice on the lower surface of an ice shelf that reaches several centimeters per annum [1]. Thus, the reaction of an ice shelf is an integral response to a change in the ocean temperature. To monitor the annual ablation/accumulation on the lower surface of the ice, one can drill an open well and set up acoustic sensors at a distance from the surface that will make it possible to make continuous observations of the dynamics of variations of the lower boundary [3]. On the one hand, this thermometer will most likely have a higher accuracy than the solely available "acoustic thermometer" [4] based on measuring the global velocity of sound in the ocean, and on the other hand, this monitoring can be carried out continuously.

The first problem arising in experiments of this type is connected with evaluating the effect of the thermal perturbation introduced by thermic drilling on the temperature and position of the lower surface of the ice shelf and evaluating the time after which one can reliably measure the ocean temperature using data on the relative position of the phase interface, i.e., when the displacement will be caused only by a change in the global ocean temperature and not by the effect of the thermic drilling. The present article is devoted to investigation of this problem.

Formulation of the Problem. Let us consider an ice layer whose upper and lower boundaries are in contact with the atmosphere and seawater, respectively (Fig. 1). We assume that the atmosphere–ice–ocean system under consideration is in thermodynamic equilibrium. In particular, the temperature at the atmosphere/ice interface is assumed to be constant, and the temperature of the ice/seawater interface and the salt concentration in the vicinity of the phase interface are related by the formula [5]

$$S^* = AT^* . \tag{1}$$

where T^* is the temperature of the phase interface; S^* is equilibrium salt concentration.

The temperature of the phase interface (the dissolution temperature) turns out to be lower than the ice melting point. We assume that a constant geothermal heat flux exists at a distance from the lower ice surface. The annual changes in the ice thickness are small and can be neglected in solving the thermal problem in the ice shelf.

At zero time, a cylindrical hole filled with fresh water at $0^{\circ}C$ is formed in the ice. The drilled well starts to freeze with time, and the surrounding ice is first heated and then cooled. The ice temperature in the region neighboring the lower portion of the well exceeds the initial equilibrium value. Therefore, the temperature of the

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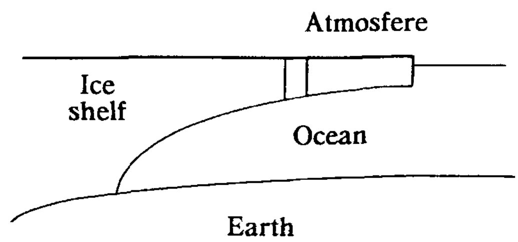


Fig. 1. Schematic representation of the problem.

ice/seawater interface increases. According to Eq. (1), the salt concentration in the water should decrease. This takes place due to dissolution of a certain amount of ice at the lower ice shelf boundary. As a result of turbulent diffusion, salt water penetrates the well region, which somewhat inhibits its freezing. The objective of this work is to evaluate the time of equilibration in the system (the relaxation time), which is an upper estimate, since we do not take into account horizontal flows of seawater under the ice shelf. Quantitative data on these flows are lacking, but it is evident that taking the flows into account can only decrease the relaxation time.

Mathematical Model and Algorithm of the Solution. We calculate the equilibrium state of the ice–seawater system prior to formation of the well. To do this, we solve the one-dimensional stationary heat-conduction equation for the region occupied by ice:

$$k_i (T_i) \frac{\partial^2 T_i}{\partial z^2} = 0 , \quad H < z < 0 , \tag{2}$$

where H is the lower ice boundary.

The upper and lower ice boundaries are kept at constant temperatures:

$$T_i (0) = T_i^0 , \tag{3}$$

$$T_i (H) = T_s^0 , \tag{4}$$

where T_i^0 and T_s^0 are the temperatures of the upper and lower ice boundaries, respectively.

The equilibrium temperature of seawater is distributed according to the law

$$k_s (T_s) \frac{\partial^2 T_s}{\partial z^2} = 0 , \quad Z < z < H . \tag{5}$$

The temperature and flux at the upper boundary of the region are equal to the corresponding quantities in the ice shelf:

$$T_s (H) = T_i (H) = T_s^0 , \tag{6}$$

$$k_s \left. \frac{\partial T_s}{\partial z} \right|_{z=H} = k_i \left. \frac{\partial T_i}{\partial z} \right|_{z=H} . \tag{7}$$

As follows from solution of systems (2)-(4) and (5)-(7), the equilibrium temperature distribution in the system is as follows:

$$T (z) = \begin{cases} \dfrac{T_s^0 - T_i^0}{H} z + T_i^0 , & H < z < 0 , \\ \dfrac{k_i}{k_s} \dfrac{T_s^0 - T_i^0}{H} z - \dfrac{k_i}{k_s} (T_s^0 - T_i^0) + T_s^0 , & Z < z < H . \end{cases} \tag{8}$$

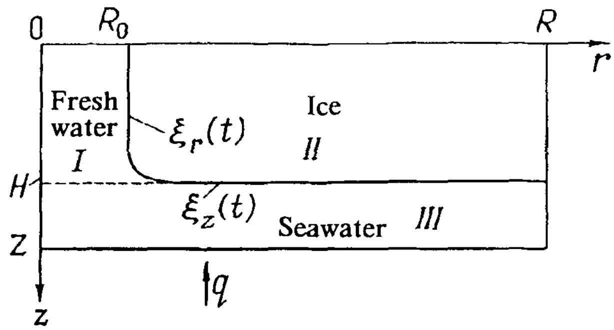


Fig. 2. Geometry of the problem.

At the instant $t = 0$, a vertical well of radius $R_0 << Z$ filled with fresh water at the temperature $T_w^0 = 0°C$ is formed (Fig. 2). The position of the phase boundary is characterized by the quantities $\xi_r(t)$ and $\xi_z(t)$. The temperature in all regions of the system is described by the equation

$$\frac{\partial T}{\partial t} C\rho (T) = \frac{1}{r} \frac{\partial}{\partial r} \left(rk(T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k(T) \frac{\partial T}{\partial z} \right),$$

$$0 < r < R, \quad H < z < Z, \quad t > 0, \tag{9}$$

$$C\rho (T) = \begin{cases} C_w \rho_w & T > T_{wi}^* \\ C_i \rho_i & T < T_{wi}^* \end{cases} \quad k(T) = \begin{cases} k_w & T > T_{wi}^* \\ k_i & T < T_{wi}^* \end{cases} \quad \text{for regions } I - II,$$

$$C\rho (T) = \begin{cases} C_s \rho_s & T > T_{si}^* \\ C_i \rho_i & T < T_{si}^* \end{cases} \quad k(T) = \begin{cases} k_s & T > T_{si}^* \\ k_i & T < T_{si}^* \end{cases} \quad \text{for regions } II - III.$$

The equality condition for temperatures and the heat-flux balance are satisfied at the boundary of the ice/water phase transition:

$$T_i (\xi_r, z, t) = T_w (\xi_r, z, t) = T_{wi}^*, \tag{10}$$

$$k_i \vec{\nabla} T |_{\vec{r} = \vec{\xi}} - k_w \vec{\nabla} T |_{\vec{r} = \vec{\xi}} = \lambda \rho \frac{d\vec{\xi}}{dt},$$

where $\vec{\nabla} T = (\partial T / \partial r)\vec{e}_r + (\partial T / \partial z)\vec{e}_z$, $\vec{\xi}(t) = (\xi_r(t), \xi_z(t))$, λ is the latent heat of freezing, and $\vec{e}_r$ and $\vec{e}_z$ are vectors of unit length.

Inasmuch as $\partial T / \partial r >> \partial T / \partial z$, the difference in heat fluxes can be represented in the following form:

$$k_i \left. \frac{\partial T}{\partial r} \right|_{r=\xi_r} - k_w \left. \frac{\partial T}{\partial r} \right|_{r=\xi_r} = \lambda \rho \frac{d\xi_r}{dt}. \tag{11}$$

The temperature is also continuous at the ice/seawater interface:

$$T_i (\xi_z, z) = T_s (\xi_z, z) = T_{si}^*, \tag{12}$$

and the heat fluxes are connected by the following relationship:

$$k_s \vec{\nabla} T |_{\vec{r} = \vec{\xi}} - k_i \vec{\nabla} T |_{\vec{r} = \vec{\xi}} = \lambda \rho \frac{d\vec{\xi}}{dt}.$$

Here $\partial T / \partial z >> \partial T / \partial r$, and the last relationship can be simplified:

$$k_s \frac{\partial T}{\partial z}\bigg|_{r=\xi_z} - k_i \frac{\partial T}{\partial z}\bigg|_{r=\xi_z} = \lambda \rho \frac{d\xi_z}{dt} . \quad (13)$$

We assume that the upper ice boundary is kept at a constant temperature:

$$T(r, 0, t) = T_i^0 , \quad 0 < r < R . \quad (14)$$

The heat flux equals zero on the well axis:

$$\frac{\partial T}{\partial r}(0, z, t) = 0 , \quad Z < z < 0 , \quad (15)$$

At sufficiently large distances, the temperature distribution remains constant and is as follows:

$$T(R, z, t) = \begin{cases} \dfrac{T_s^0 - T_i^0}{H} z + T_i^0 , & H < z < 0 , \\ \dfrac{k_i}{k_s} \dfrac{T_s^0 - T_i^0}{H} z - \dfrac{k_i}{k_s}(T_s^0 - T_i^0) + T_s^0 , & Z < z < H . \end{cases} \quad (16)$$

and the constant heat flux

$$q(r, Z, t) = k_i \frac{T_s^0 - T_i^0}{H} . \quad (17)$$

is supplied to the lower boundary of the region under consideration. We write the initial conditions:

$$T(r, z, 0) = T_w^0 , \quad 0 < r < R_0 , \quad H < z < 0 , \quad (18)$$

$$T(r, z, 0) = T(z) , \quad R_0 < r < R , \quad Z < z < 0 \quad \text{and} \quad 0 < r < R_0 , \quad Z < z < H , \quad (19)$$

$$\xi_r(0) = R_0 , \quad \xi_z(0) = H ,$$

where T_w^0 is the initial water temperature in the well.

Prior to the instant of freezing, diffusion of seawater into the newly formed hole takes place. The distribution of the salt concentration is described by the diffusion equation

$$\frac{\partial S}{\partial t} = D_S \left(\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial S}{\partial r} \right) + \frac{\partial}{\partial z}\left(\frac{\partial S}{\partial z} \right) \right) ,$$

$$0 < r < R , \quad Z < z < \xi_z \quad \text{and} \quad 0 < z < \xi_r , \quad H < z < 0 . \quad (20)$$

The balance relationship for the salt mass is satisfied at the ice/seawater interface:

$$- D_S \frac{\partial S}{\partial z}\bigg|_{r=\xi_z} = S^* \frac{d\xi_z}{dt} , \quad S^* = A T_{si}^*(r, \xi_z, t) . \quad (21)$$

The following conditions exist at the boundaries of the region $0 < r < R$, $Z < z < 0$:

$$S(r, 0, t) = 0 , \quad (22)$$

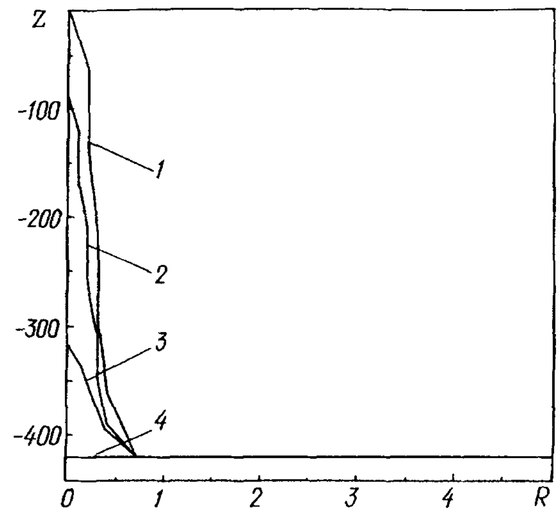


Fig. 3. Phase-interface profile at different instants: 1) 20 hours, 2) 1.6 days, 3) 4, 4) 16. Z, R, m.

$$\frac{\partial S}{\partial r}(0, z, t) = 0\,, \tag{23}$$

$$S\,(r, Z, t) = S_0\,, \tag{24}$$

$$S\,(R, z, t) = \begin{cases} 0\,, & H < z < 0\,, \\ S_0\,, & Z < z < H\,. \end{cases} \tag{25}$$

The initial conditions are as follows:

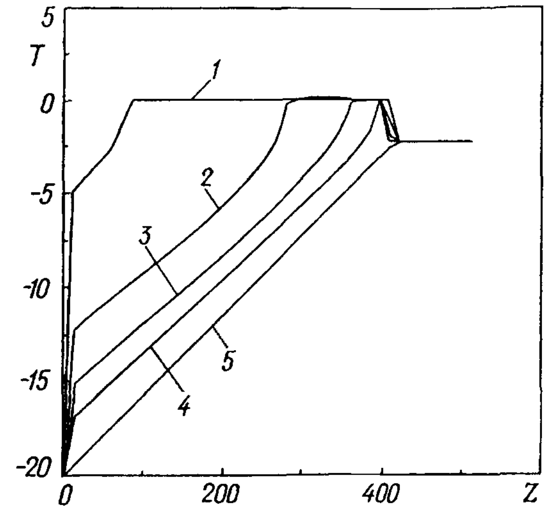
$$S\,(r, z, 0) = \begin{cases} 0\,, & 0 < r < R\,, \quad H < z < 0\,, \\ S_0\,, & 0 < r < R\,, \quad Z < z < H\,. \end{cases} \tag{26}$$

Problem (9)-(26) was solved numerically using the alternating-directions method [6, 7], whose essence is as follows. The transition from one temporal layer to another is carried out in two stages, which makes it possible to write difference operators for derivatives over r and z for different temporal layers. The obtained equations are solved by means of the sweep method first along the z direction and then along the r direction. The implicit difference scheme of this method has an approximation order of $O(h^2 + \tau^2)$, where h and τ are the maximum coordinate step and the time step, respectively.

Calculations were carried out for the following values of the parameters: $C_w = 4.181$ kJ/kg·deg, $C_i = 2.3$ kJ/kg·deg, $C_s = 4.061$ kJ/kg·deg, $\rho_w = 1000$ kg/m^3, $\rho_i = 917$ kg/m^3, $\rho_s = 1033$ kg/m^3, $k_w = 0.561$ J/m·sec·deg, $k_i = 2.2$ J/m·sec·deg, $k_s = 647$ J/m·sec·deg, $T_i^0 = -20$°C, $T_w^0 = 0$°C, $T_s^0 = -2.14$°C, $\lambda = 334$ kJ/kg, $S_0 = 0.0258$, $A = -0.0154$, $R = 50$ m, $Z = 510$ m, and $H = 420$ m.

Results and Discussion. The profile in the well at different instants is shown in Fig. 3. Inasmuch as the temperature of the upper portion of the ice shelf is substantially lower than that at the interface with seawater, the upper portion freezes at a higher rate. Complete freezing takes place in approximately 16 days.

Let us consider the temperature variations in the ice shelf induced by the well. Upon formation of the well, the temperature of the surrounding ice increases, so that a temperature wave propagates along the radial direction. Temperature profiles along the well axis and the corresponding radial profiles are presented in Figs. 4 and 5, respectively. It is evident that temperature relaxation proceeds rather rapidly (in ~15 days). It should be noted, however, that complete equilibration of the temperature to its initial value takes about 10 years.



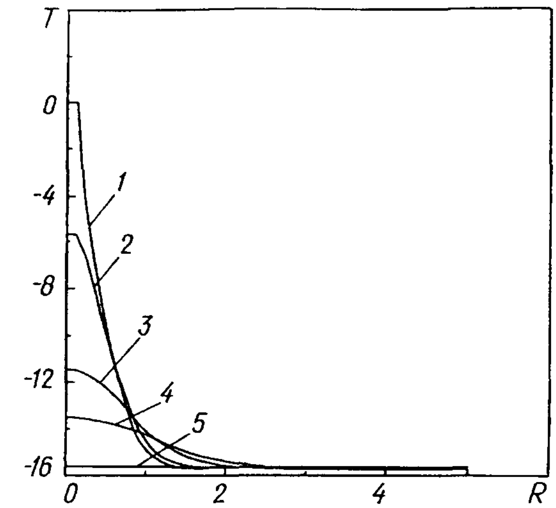
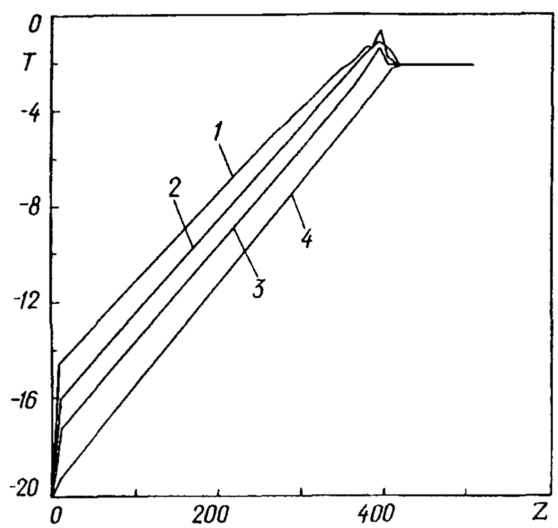


Fig. 4. Time dependence of the temperature at the well axis: 1) 1.6 days, 2) 3.5, 3) 6, 4) 10, 5) 1 year. T, °C.

Fig. 5. Temperature distribution over the radius at Z = 100 m at different instants: 1) 1.6 days, 2) 2, 3) 5, 4) 10, 5) 1 year.



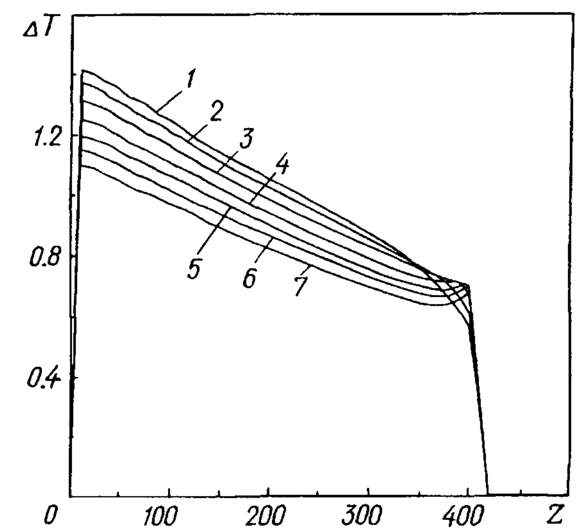


Fig. 6. Temperature at a distance of 0.5 m from the well axis at different instants: 1) 1.6 days, 2) 6, 3) 10, 4) 21.

Fig. 7. Deviation of the temperature from the equilibrium value at a distance of 1 m at different instants of time: 1) 8 days, 2) 9, 3) 10.5, 4) 10, 5) 13, 6) 14.5, 7) 16.

An increase in the temperature in the vicinity of the ice/ocean interface leads to dissolution of ice. This leads to release of energy, which is supplied to the upper portion of the ice shelf. Inasmuch as the coefficients of turbulent diffusion and thermal diffusivity of seawater are much greater than the thermal diffusivity of ice, dissolution accompanied by energy release is more intense than the heat transfer to ice. As a result, a nonmonotonic portion is observed on the temperature profile in the vicinity of the phase interface (Fig. 6). This nonmonotonic behavior is also observed for the deviation of the temperature from the initial equilibrium value (Fig. 7). The temperature profile approaches the equilibrium state with time. No considerable thermal perturbation is observed at distances exceeding 16 m, whereas dissolution of the lower portion of the ice shelf takes place at distances not exceeding 9 m.

CONCLUSIONS

1. The temperature regime of an ice shelf in the vicinity of a hot water-drilled well is investigated.

2. Dissolution of the lower surface of the ice shelf takes place at distances not exceeding ~ 9 m. Relaxation to the steady state takes place in ~ 16 days.

3. The temperature of the lower surface of the ice shelf approaches a state close to equilibrium in ~ 15 days, although a small perturbation persists for as long as 10 years.

4. Monitoring of the global ocean temperature based on observations of the dissolution of the lower surface of the ice shelf can be carried out starting from the instant $t = 10$ days (for $r \sim 2$ m) ($t = 6$ days for $r = 3$ m, and virtually any instant can be chosen for $r > 5$ m), since, starting from this instant, deviations of the lower surface of the ice shelf do not exceed 1 mm, which is substantially smaller than the expected variations due to an increase or decrease in the global ocean temperature.

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NOTATION

r and z, coordinates; t, time; R and Z, boundaries of the region under consideration; R_0, diameter of the hole; H, thickness of the ice layer; ξ_r and ξ_z, radial and vertical components of the phase interface; $T(r, z, t)$, temperature at the point with coordinates (r, z) at the instant t; T_i, T_w, and T_s, temperature of ice, fresh water, and seawater, respectively; T_i^0, T_w^0, and T_s^0, initial temperature of ice, fresh water, and sea water, respectively; T_{wi}^* and T_{si}^*, temperature of the water–ice and ice–seawater phase transitions, respectively; C, specific heat; C_i, C_w, and C_s, specific heat of ice, fresh water, and sea water, respectively; ρ, density; ρ_i, ρ_w, and ρ_s, density of ice, fresh water, and seawater, respectively; k, thermal conductivity; k_i, k_w, and k_s, thermal conductivity of ice, fresh water, and sea water, respectively; λ, latent heat of solidification (dissolution); $S(r, z, t)$ and S_0, current and initial salt concentrations in seawater; S^*, equilibrium salt concentration at ice/seawater interface; D_S, diffusion coefficient; A, proportionality factor in the dependence of the equilibrium salt concentration on the temperature at the ice/seawater interface.

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